

สมบัติของลอการิทึม

$$1. \log_a 1 = 0 \quad (\text{เพราะ } a^0 = 1)$$

$$2. \log_a a = 1 \quad (\text{เพราะ } a^1 = a)$$

$$3. \log_a a^x = x \quad \text{และ} \quad a^{\log_a x} = x$$

$$\text{ถ้า } \log_a a^x = \square$$

$$a^x = a^\square$$

$$\text{ถ้า } \square = x$$

$$\therefore \log_a a^x = x$$

$$\text{ถ้า } \log_a x = \square$$

$$x = a^\square$$

$$x = a^{\log_a x}$$

$$\therefore a^{\log_a x} = x$$

$$4. \text{ ถ้า } \log_a x = \log_a y \text{ แล้ว } x = y$$

$$\text{ถ้า } \log_a x = u \quad \text{และ} \quad \log_a y = v$$

$$x = a^u$$

$$y = a^v$$

$$\text{เนื่องจาก } u = v \quad \text{ถ้า } a^u = a^v$$

$$\text{ดังนั้น } x = y$$

$$5. \log_a x = \frac{\log_b x}{\log_b a}$$

$$\text{ถ้า } \log_a x = y$$

$$x = a^y$$

$$\log_b x = \log_b a^y$$

$$\log_b x = y \cdot \log_b a$$

$$y = \frac{\log_b x}{\log_b a}$$

$$\therefore \log_a x = \frac{\log_b x}{\log_b a}$$

$$6. \log_a (u \cdot v) = \log_a u + \log_a v$$

$$\text{Let } \log_a u = x \quad \text{nm: } \log_a v = y$$

$$u = a^x$$

$$v = a^y$$

$$\therefore u \cdot v = a^x \cdot a^y = a^{x+y}$$

$$\log_a u \cdot v = \log_a a^{x+y}$$

$$= (x+y) \log_a a$$

$$= x+y$$

$$\therefore \log_a u \cdot v = \log_a u + \log_a v$$

$$7. \log_a \left(\frac{u}{v} \right) = \log_a u - \log_a v$$

$$\text{Let } \log_a u = x \quad \text{nm: } \log_a v = y$$

$$u = a^x$$

$$v = a^y$$

$$\therefore \frac{u}{v} = \frac{a^x}{a^y} = a^{x-y}$$

$$\log_a \frac{u}{v} = \log_a a^{x-y}$$

$$= (x-y) \log_a a$$

$$= x-y$$

$$\therefore \log_a \frac{u}{v} = \log_a u - \log_a v$$

$$8. \log_a u^n = n \log_a u$$

$$\text{Let } \log_a u = x$$

$$u = a^x$$

$$u^n = (a^x)^n = a^{nx}$$

$$\therefore \log_a u^n = \log_a a^{nx}$$

$$= nx \cdot \log_a a$$

$$\therefore \log_a u^n = nx = n \log_a u$$

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$$9. \log_a b = \frac{1}{\log_b a}$$

$$\text{ถ้า } \log_a b = x$$

$$\frac{\log_b b}{\log_b a} = x$$

$$\therefore x = \frac{1}{\log_b a}$$

$$10. \log_{a^n} b = \frac{1}{n} \log_a b, n \neq 0$$

$$\text{ถ้า } \log_{a^n} b = \frac{\log b}{\log a^n}$$

$$= \frac{\log b}{n \log a}$$

$$= \frac{1 \cdot \log b}{n \log a}$$

$$\therefore \log_{a^n} b = \frac{1}{n} \log_a b$$

$$11. a^{\log_c b} = b^{\log_c a}$$

$$\text{ถ้า } a^{\log_c b} = x$$

$$\text{ถ้า } \log_a a^{\log_c b} = \log_a x$$

$$(\log_c b)(\log_c a) = \log_c x$$

$$(\log_c a)(\log_c b) = \log_c x$$

$$\log_c b^{\log_c a} = \log_c x$$

$$b^{\log_c a} = x = a^{\log_c b}$$

$$\therefore a^{\log_c b} = b^{\log_c a}$$

$$12. \log_{ab} a + \log_{ab} b = 1$$

$$\text{เนื่องจาก } \log_{ab} a + \log_{ab} b = \log_{ab} (ab)$$

$$= 1_{ab}$$

$$\therefore \log_{ab} a + \log_{ab} b = 1$$